COMPARISON OF DIFFERENCING PARAMETER ESTIMATION
FROM NONSTATIONARY ARFIMA MODEL BY GPH METHOD
WITH COSINE TAPERING

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ABSTRACT

This paper compares the accuracy of estimation of differencing parameter from nonstationary ARFIMA data. For comparison purposes, the GPH (Geweke and Porter-Hudak) method is modified by three tapers, there are Cosine bell, Hanning and Hamming tapers. Accuracy among three methods are justified by Mean Square Error and Deviation Standard in two models, i.e ARFIMA(1,d,0) and ARFIMA(0,d,1). From simulation results, GPH method with Cosine Bell tapering shows a good performance in estimating the differencing parameter of ARFIMA(0,d,1) data. From ARFIMA(1,d,0) data, GPH method with Hanning tapering is the best of all methods.

Keywords: ARFIMA, Periodogram, Taper

1. INTRODUCTION

Long range dependence or long memory means that observations far away each other are still strongly correlated. The correlation of a long-memory process decay slowly that is with a hyperbolic rate, not exponentially like for example ARMA-process (see figure.1).

The literature on ARFIMA processes has rapidly increased since early contribution by Granger and Joyeux (1980), Hosking (1981) and Geweke and Porter-Hudak (1983). This theory has been widely used in different fields such as meteorology, astronomy, hydrology and economics.

Geweke and Porter-Hudak (1983) presented a very important work on stationary long memory processes. Their paper gave rise to several other works, and presented a proof for the asymptotic distribution of the long memory parameter. These authors proposed an estimator of $d$ as the ordinary least squares estimator of the slope parameter in a simple linear regression of the logarithm of the periodogram. Reisen (1994) proposed a modified form of the regression method, based on a smoothed version of the periodogram function. Robinson (1995) has made a mild modifications on Geweke and Porter-Hudak’s estimator, dealt simultaneously with $d \in (-0.5,0.0)$ and $d \in (0.0,0.5)$. Hurvich and Deo (1999), among others, addressed the problem of selecting the number of frequencies necessary for estimating the differencing parameter in the stationary case. Fox and Taquq (1986) considered an approximation method, whereas Sowell (1992) presented the exact maximum likelihood procedure for estimating the fractional parameter. These two papers considered the estimation procedures for the stationary case. Simulation studies comparing estimates of $d$ may be found, for instance, in Bisaglia and Guegan (1998), Reisen and Lopes (1999).

Here, by simulation study, we compare accuracy of estimation the differencing parameter of nonstationary ARFIMA model by the spectral regression methods with cosine tapered periodogram. The first estimator is the modified periodogram function where the tapered data is obtained from the cosine-bell function, this estimator was used in the works by Hurvich and Ray (1995) and Velasco (1999a). As the second and
the third estimator, we consider Hanning and Hamming tapers respectively for estimating the differencing parameter of ARFIMA model. Autoregressive and Moving average parameters are estimated by maximum likelihood.

![Series data long memory](image)

Figure 1. ACF plot of long memory process

2. ARFIMA MODEL

A well known class of long memory models is the autoregressive fractionally integrated moving average (ARHMA) process introduced by Granger and Joyeux (1980) and Hosking (1981).

An ARFIMA model \((p,d,q)\) can be defined as follows:

\[
\phi(B) (1 - B)^d \{ \xi \gamma - \mu \} \theta(B) a_t
\]

where

- \(t\) = index of observation \((t = 1, 2, ..., T)\)
- \(d\) = the degree of differential parameter \((\text{real number})\)
- \(\mu\) = mean of observation
- \(\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p\), polynomial of AR\((p)\)
- \(\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q\), polynomial of MA\((q)\)
- \((1 - B)^d\) = fractional differencing operator
- \(a_t\) is IID\((0, \sigma^2)\).

3. GPH METHOD

There have been proposed in the literature many estimators for the fractional differencing parameter. We shall concentrate in estimators based upon the estimation of the spectral density function. The semi-parametric estimators describe below are obtained taking the logarithm of the spectral density. Estimation of \(d\) from ARFIMA \((p,d,q)\) model as follows

a) Construct spectral function of ARFIMA \((p,d,q)\) model

For the ARFIMA model given in equation (1), let \(w_t = (1 - B)^d z_t\) and let \(f_w(\omega)\) and \(f_z(\omega)\) be the spectral density function of \(\{w_t\}\) and \(\{z_t\}\), respectively. Then,

\[
f_z(\omega) = f_w(\omega) \left\{ 2 \sin(\omega/2) \right\}^{-2d}, \quad 0 \leq \omega \leq \pi
\]

Where
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\[ f_W(\omega_j) = \frac{\sigma^2}{2\pi} \left| \theta_q \left( \exp(-i\omega_j) \right) \right|^2, \]

is the spectral density of a regular ARMA \((p,q)\) model. Note that \( f_Z(\omega) \to \infty \) as \( \omega \to 0 \).

b) Take logarithms on both sides of equation (2).

\[
\ln \left\{ f_Z(\omega_j) \right\} = \ln \left\{ \frac{f_W(\omega_j)}{f_W(0)} \right\} = \ln f_W(0) + \ln \left\{ \exp(i\omega_j) \right\} - \ln \left\{ \exp(-i\omega_j) \right\}
\]

\[
= \ln f_W(0) + d\ln \left\{ \exp(i\omega_j) \right\} - 2\ln \left\{ \frac{f_W(\omega_j)}{f_W(0)} \right\}
\]

(3)

c) Add \( \ln I_Z(\omega_j) \), the natural logarithm of periodogram \( \{Z_t\} \) to Both sides of equation (3) above,

\[
\ln \left\{ I_Z(\omega_j) \right\} = \ln \left\{ \frac{f_W(\omega_j)}{f_W(0)} \right\} = \ln \left\{ \exp(\omega_j) \right\} - \ln \left\{ \exp(-\omega_j) \right\}
\]

\[
= \ln f_W(0) + \ln \left\{ \exp(i\omega_j) \right\} - 2\ln \left\{ \frac{f_W(\omega_j)}{f_W(0)} \right\}
\]

(4)

d) Determine the periodogram from equation (4)

Geweke and Porter-Hudak (1983) obtained periodogram by

\[
I_Z(\omega_j) = \frac{1}{2\pi} \left\{ \sum_{t=0}^{T-1} \exp(t\omega_j) \right\}, \omega_j = -\pi(\pi).
\]

(5)

Hurvich and Ray (1995) and Velasco(1999a) used modified periodogram function by

\[
I_Z(\omega_j) = \frac{1}{2\pi} \left\{ \sum_{t=0}^{T-1} \exp(t\omega_j) \right\}^2
\]

(6)

where the tapered data is obtained from the cosine-bell function

\[
tap(t) = \frac{1}{2} \left[ 1 - \cos \left( \frac{2\pi (t + 0.5)}{T} \right) \right].
\]

For Comparison, we use another tapers, there are Hanning taper \( tap(t) = \frac{1}{2} \left[ 1 - \cos \left( \frac{2\pi (t + 1)}{T} \right) \right] \) and Hamming taper \( tap(t) = 0.54 - 0.46 \cos \left( \frac{2\pi t}{T-1} \right) \).

e) Estimate the differencing parameter \( d \)

From equation (4), for \( \omega_j \) near zero, i.e., for \( j = 1, \ldots, m << (T/2) \) such that \( m/T \to 0 \) as \( T \to \infty \), we have \( \ln \left\{ f_W(\omega_j) \right\} \approx 0 \). Thus,

\[
Y_j = \beta_0 + \beta X_j + \alpha_j, \quad j = 1, 2, 3, \ldots, m
\]

\( \hat{\beta}_1 = \hat{d} \) could be estimated by Ordinary Least Square (OLS). For computation, with Euler equation we have

\[
Y_j = \ln I_Z(\omega_j) = \ln \left( \frac{1}{4\sin^2(\omega f^2)} \right)
\]

After determining the value of \( d \), we estimate Autoregressive (AR) and moving average (MA) parameters. Autoregressive \( (\phi) \) and Moving average \( (\theta) \) parameters are estimated by maximum likelihood method (Sowell, 1992).
4. SIMULATION STUDY

We have conducted simulation studies to obtain some information about the performance of the accuracy of spectral regression methods in estimating the degree of differencing parameter from ARFIMA model. In this simulation study we use three estimators five methods of spectral regression methods namely Cosine Bell, Hanning and Hamming tapers. The simulation is done for considering $T = 300, 600$ and $1000$ serial data with 1000 replications. In this study, time series data are generated according to the particular specification Generate ARFIMA (1,$d$,0) and ARFIMA(0,$d$,1) models for simulated data, with $T = 600$ and $T = 1000$, respectively. Both types of data above have specification as follow $a_i \sim N(0,1)$, $(\phi, \theta) = 0.5$, $d = 0.6, 0.7$ and 0.8.

For each series, we estimate the value of $d$ through the three methods above and later we take the arithmetic average and standard deviation (SD) of these values.

After estimating ARFIMA parameters, we calculate Mean square error of forecasting with the value of out sample $h = 10$. MSE of forecasting could be seen at table 2.

Table 1 Mean and Standard deviation of parameter estimation $d$ from ARFIMA model data.

<table>
<thead>
<tr>
<th>T</th>
<th>ARFIMA Model Data</th>
<th>GPH Estimation Method With Cosine Taper</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Cosine Bell</td>
</tr>
<tr>
<td></td>
<td></td>
<td>d</td>
</tr>
<tr>
<td>600</td>
<td>ARFIMA(1,$d$,0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$d=0.6$</td>
<td>0.624</td>
</tr>
<tr>
<td></td>
<td>$d=0.7$</td>
<td>0.717</td>
</tr>
<tr>
<td></td>
<td>$d=0.8$</td>
<td>0.840</td>
</tr>
<tr>
<td></td>
<td>ARFIMA(0,$d$,1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$d=0.6$</td>
<td>0.581</td>
</tr>
<tr>
<td></td>
<td>$d=0.7$</td>
<td>0.687</td>
</tr>
<tr>
<td></td>
<td>$d=0.8$</td>
<td>0.791</td>
</tr>
<tr>
<td>1000</td>
<td>ARFIMA(1,$d$,0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$d=0.6$</td>
<td>0.611</td>
</tr>
<tr>
<td></td>
<td>$d=0.7$</td>
<td>0.712</td>
</tr>
<tr>
<td></td>
<td>$d=0.8$</td>
<td>0.803</td>
</tr>
<tr>
<td></td>
<td>ARFIMA(0,$d$,1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$d=0.6$</td>
<td>0.598</td>
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<tr>
<td></td>
<td>$d=0.7$</td>
<td>0.695</td>
</tr>
<tr>
<td></td>
<td>$d=0.8$</td>
<td>0.803</td>
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</table>
Table 2 MSE of Forecasting ($h = 10$)

<table>
<thead>
<tr>
<th>T</th>
<th>ARFIMA Model Data</th>
<th>Cosine Taper</th>
<th>Hanning</th>
<th>Hamming</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>ARFIMA(1, $d$, 0)</td>
<td>$d=0.6$</td>
<td>1.316</td>
<td>1.317</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d=0.7$</td>
<td>1.322</td>
<td>1.322</td>
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<tr>
<td></td>
<td></td>
<td>$d=0.8$</td>
<td>1.342</td>
<td>1.342</td>
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<tr>
<td></td>
<td>ARFIMA(0, $d$, 1)</td>
<td>$d=0.6$</td>
<td>1.254</td>
<td>1.254</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d=0.7$</td>
<td>1.254</td>
<td>1.255</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d=0.8$</td>
<td>1.232</td>
<td>1.233</td>
</tr>
<tr>
<td>600</td>
<td>ARFIMA(1, $d$, 0)</td>
<td>$d=0.6$</td>
<td>1.367</td>
<td>1.367</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d=0.7$</td>
<td>1.323</td>
<td>1.323</td>
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<tr>
<td></td>
<td></td>
<td>$d=0.8$</td>
<td>1.352</td>
<td>1.352</td>
</tr>
<tr>
<td></td>
<td>ARFIMA(0, $d$, 1)</td>
<td>$d=0.6$</td>
<td>1.262</td>
<td>1.262</td>
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<tr>
<td></td>
<td></td>
<td>$d=0.7$</td>
<td>1.219</td>
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<tr>
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<td></td>
<td>$d=0.8$</td>
<td>1.633</td>
<td>1.633</td>
</tr>
</tbody>
</table>

5. CONCLUSION

From simulation results, GPH method with Cosine Bell tapering shows a good performance in estimating the differencing parameter of ARFIMA(0, $d$, 1) data. From ARFIMA(1, $d$, 0) data, GPH method with Hanning tapering is the best of all methods. From forecasting result, Mean square error of GPH method with Hamming tapering has the least value of all data types.

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REFERENCES


