COMPARISON OF THE DIFFERENCING PARAMETER ESTIMATION FROM ARFIMA MODEL BY SPECTRAL REGRESSION METHODS

By

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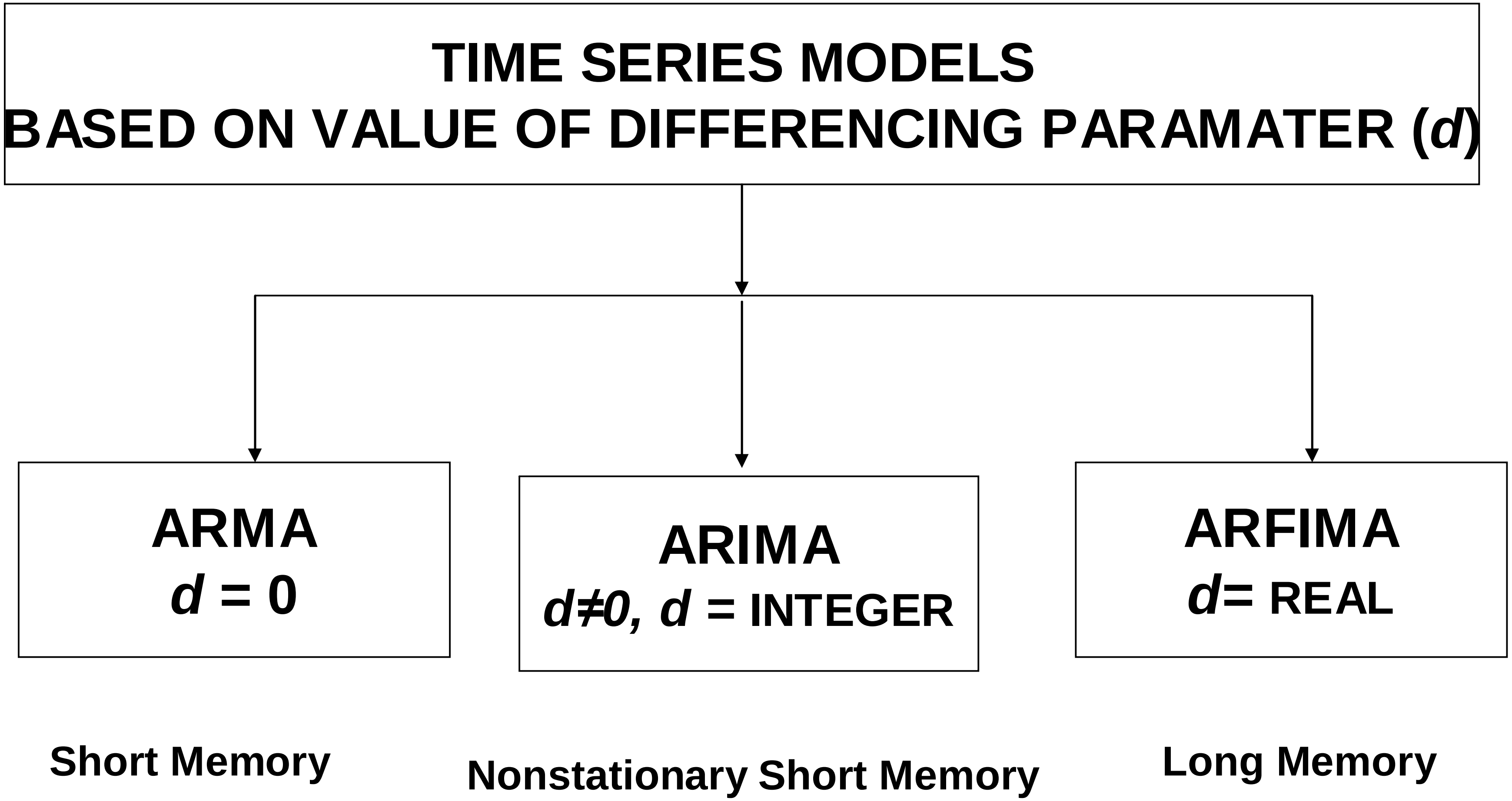
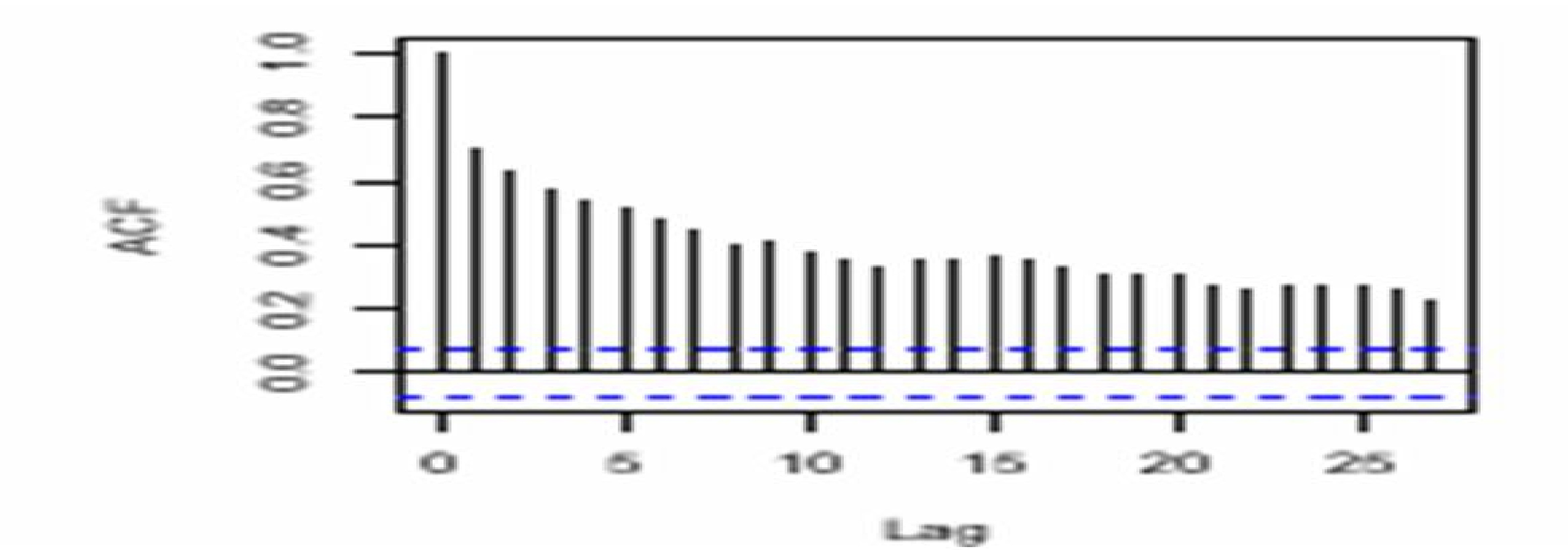
INTRODUCTION (1)

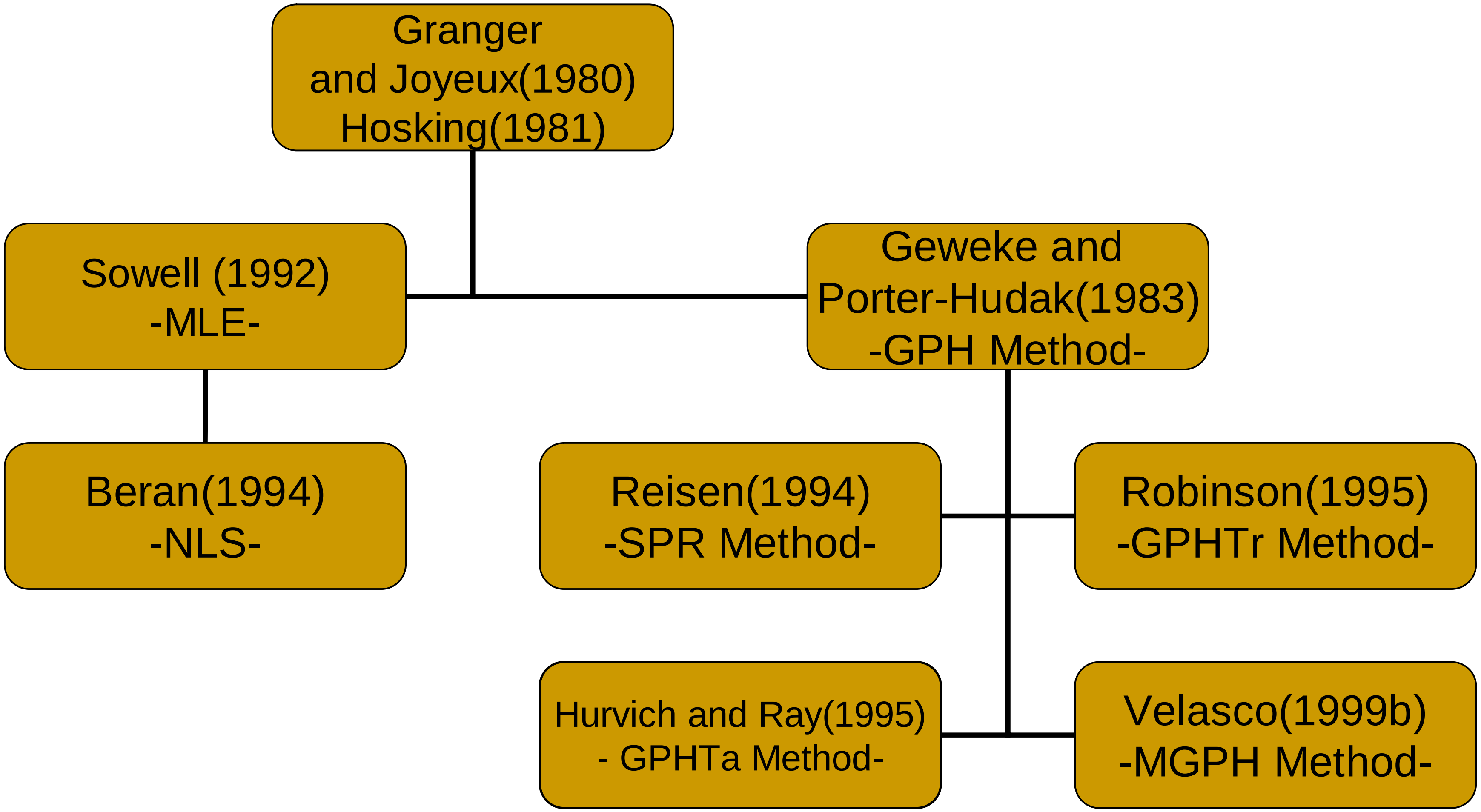


Long Memory Processes

- Long range dependence or long memory means that observations far away from each other are still strongly correlated.
- The correlation of long memory processes is decay slowly as lag data increase that is with a hiperbollic rate.



INTRODUCTION(2)



INTRODUCTION(3)

COMPARISON OF
REGRESSION SPECTRAL
METHODS

Lopes,Olberman
and Reisen(2004)
-Non stationary ARFIMA-
SPR Method is the best

Lopes
and Nunes(2006)
-ARFIMA(0,*d*,0)-
GPH Method is the best

GOAL OF RESEARCH

Comparing accuracy of spectral regression estimation methods of the differencing parameter (*d*) from stationary ARFIMA Model by Simulation Study.

ARFIMA MODEL(1)

- An ARFIMA(*p,d,q*) model can be defined as follows:

$$\phi(B)(\;-\;))^{d}(1_{t}\;)\quad B\quad(\;)\quad z$$

- t = index of observation (t = 1, 2, . . ., T)
- d = differencing parameter (real number)
- μ = mean of obervation

$$a_t \sim NID(0, \sigma^2),$$

ARFIMA MODEL(2)

- $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p$ is polinomial AR(p)
- $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q$ is polinomial MA(q)
- $(1-B)^d = \nabla^d = \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k$ fractional differencing operator

Additive Outlier in Time Series Data

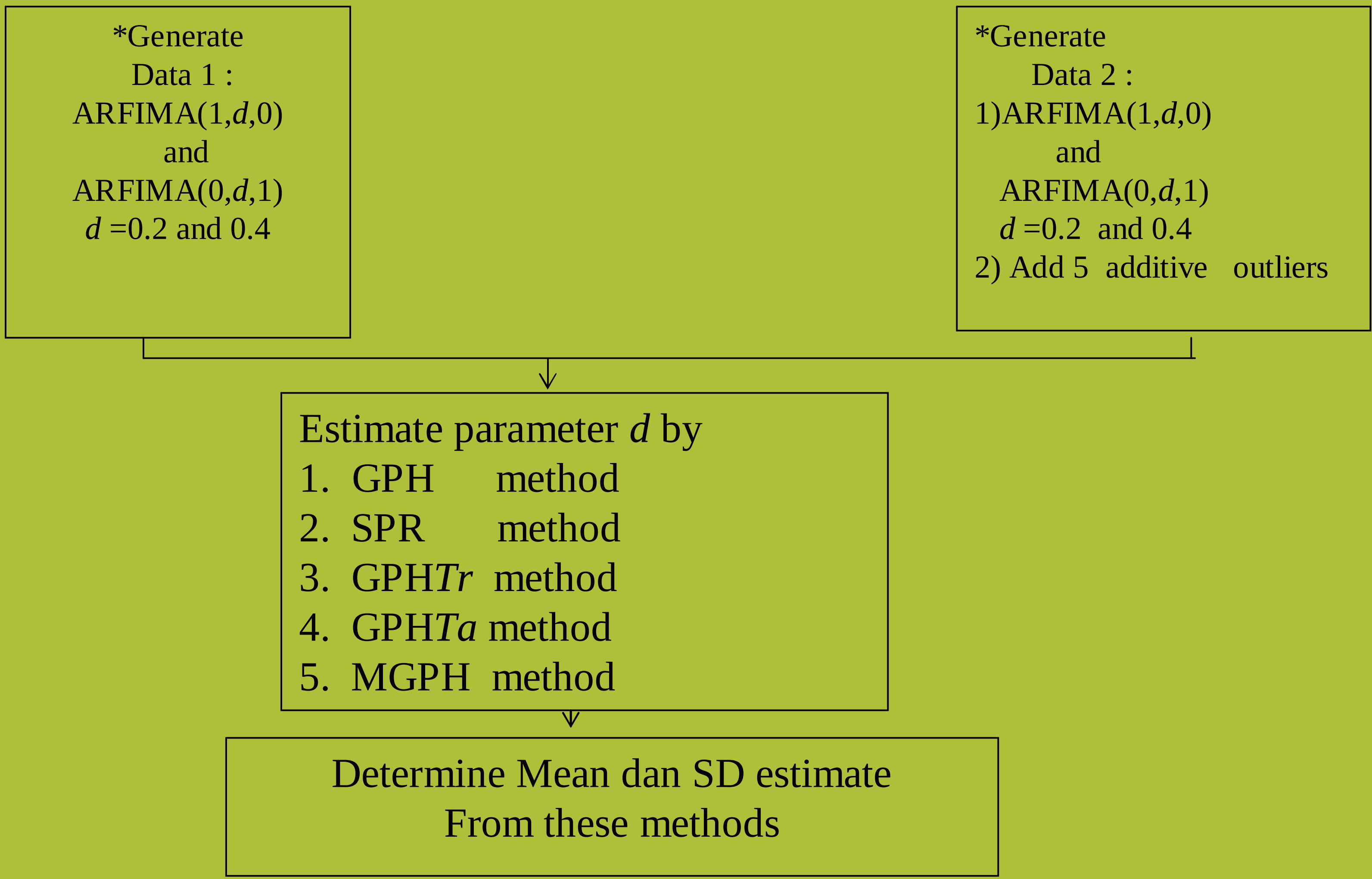
Additive Outlier is an event that effects a series for one time period only

$$Z_t = \begin{cases} X_t & t \neq T \\ X_t + \omega_{AO} & t = T \end{cases}$$

$$= X_t + \omega_{AO} I_t^{(T)}$$

$$AO : \lambda_{1,T} = \hat{}$$

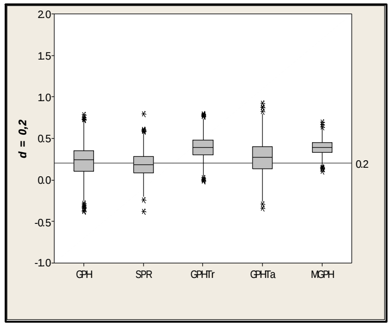
DIAGRAM


*Generate
Data 1 :
ARFIMA(1,*d*,0)
and
ARFIMA(0,*d*,1)
d =0.2 and 0.4
*Generate
Data 2 :
1)ARFIMA(1,*d*,0)
and
ARFIMA(0,*d*,1)
d =0.2 and 0.4
2) Add 5 additive outliers
Estimate parameter *d* by
1. GPH method
2. SPR method
3. GPH*Tr* method
4. GPH*Ta* method
5. MGPH method
Determine Mean dan SD estimate
From these methods

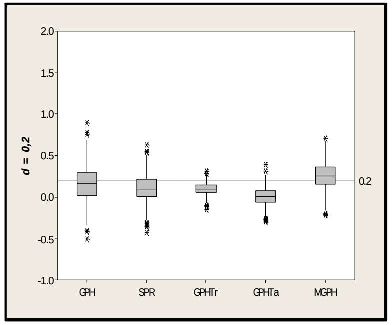
COMPARISON RESULT (1)

$d = 0.2$

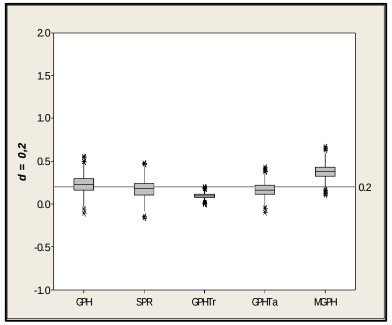
Without Outliers



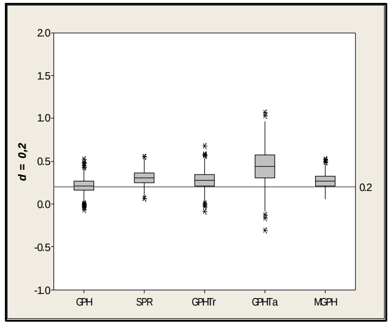
ARFIMA(1,*d*,0), T= 300



ARFIMA(0,*d*,1), T =300

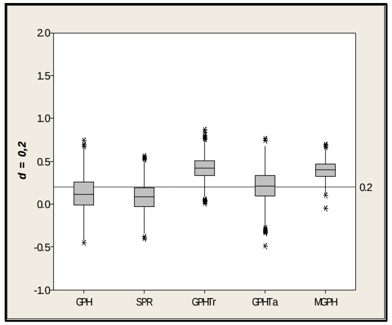


ARFIMA(1,*d*,0),T =1000

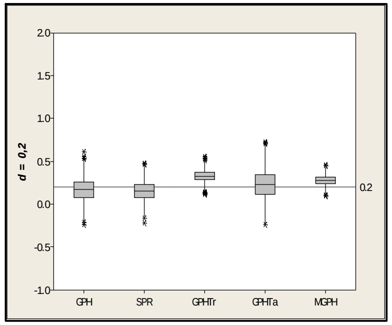


ARFIMA(0,*d*,1),T =1000

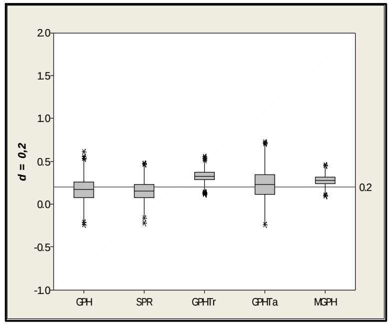
With Outliers



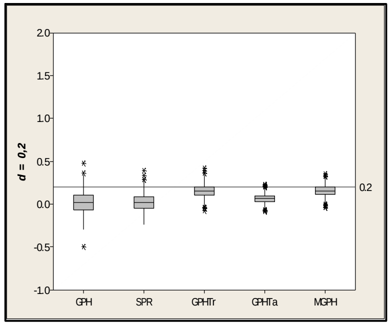
ARFIMA(1,*d*,0), T= 305



ARFIMA(0,*d*,1), T= 305



ARFIMA(1,*d*,0),T =1005

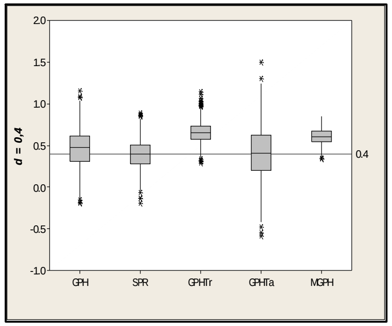


ARFIMA(0,*d*,1), T =1005

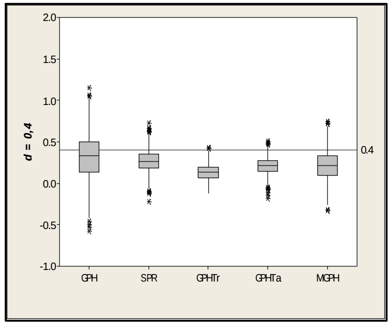
COMPARISON RESULT (2)

d = 0.4

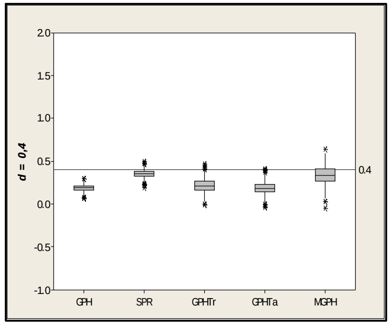
Without Outliers



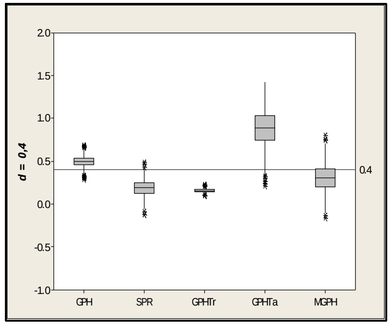
ARFIMA(1,*d*,0), T= 300



ARFIMA(0,*d*,1), T =300

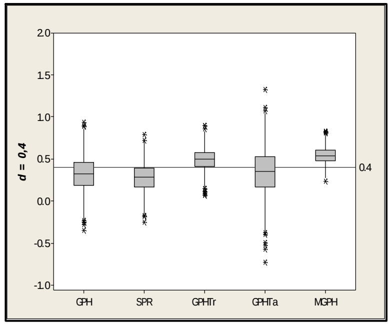


ARFIMA(1,*d*,0),T =1000

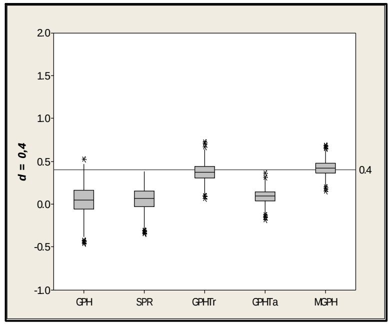


ARFIMA(0,*d*,1),T =1000

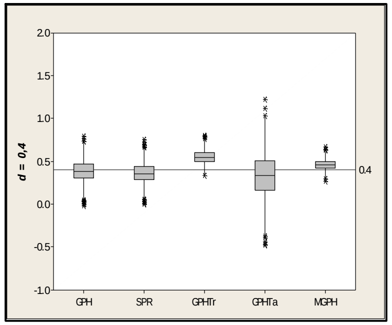
With Outliers



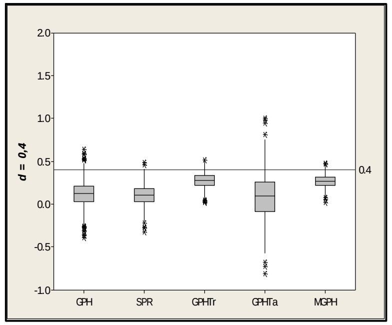
ARFIMA(1,*d*,0), T= 305



ARFIMA(0,*d*,1), T= 305



ARFIMA(1,*d*,0),T =1005



ARFIMA(0,*d*,1), T =1005

CONCLUSION

1) GPH method shows the best performance in estimating the differencing parameter with value *d* = 0.2 of ARFIMA model for both clean data and data with outlier.
2) Estimation of spectral regression methods are better to be implemented for modeling ARFIMA(1,*d*,0) data than for modeling ARFIMA(0,*d*,1) data.

THANK YOU…..

THANK YOU

ESTIMATION OF THE DIFFERENCING PARAMETER OF ARFIMA MODEL BY SPECTRAL REGRESSION METHOD(1)

1. Construct spectral density function (SDF) of ARFIMA model

$$f_Z(\omega)=\frac{\sigma_a^2}{2\pi}\left|\frac{\theta_q(\exp(-i\omega))}{\phi_p(\exp(-i\omega))}\right|^2\left\{2\sin\left(\frac{\omega}{2}\right)\right\}^{-2d}, \quad \omega\in[-\pi,\pi], \quad (2)$$

2. Take logarithms of SDF from ARFIMA model

$$\ln f_Z(\omega_j)=\ln f_W(0)+d\ln\left|1-\exp(-i\omega_j)\right|^{-2}+\ln\left(\frac{f_W(\omega_j)}{f_W(0)}\right) \quad (3)$$

where $\omega_j=\frac{2\pi j}{T}$, $j=1,2,\ldots,[T/2]$

ESTIMATION OF THE DIFFERENCING PARAMETER OF ARFIMA MODEL BY SPECTRAL REGRESSION METHOD(2)

3. Add natural logarithm of periodogram to equation (3) above

$$\ln\left\{I_Z(\omega_j)\right\} = \ln\left\{f_W(0)\right\} - d\ln\left\{\left|1-\exp(-i\omega_j)\right|^2\right\} + \ln\left\{\frac{f_W(\omega_j)}{f_W(0)}\right\} + \ln\left\{\frac{I_Z(\omega_j)}{f_Z(\omega_j)}\right\} \quad (4)$$

4. Determine the periodogram based on regression spectral methods

GPH, GPH*tr*, MGPH

$$I_Z(\omega_j) = \frac{1}{2\pi}\left\{\gamma_0 + 2\sum_{t=1}^{g(T)} \gamma_t \cos(t.\omega_j)\right\},$$

ESTIMATION OF THE DIFFERENCINGPARAMETER OF ARFIMA MODEL BY SPECTRAL REGRESSION METHOD(3)

SPR

$$I_Z(\omega_j) = \frac{1}{2\pi}\left\{ \sum_{t=1}^{g^*(T)} \gamma \; \tau \; + \quad {}_t \right\}_t \quad (\; 2 \; \tau_j)$$

$$\tau_t = \begin{cases} 1 - (6 \quad t \; / \; g^*(T))^2 \quad (6 \quad t \; / \; g^*(T))^3 \quad \frac{g^*(T)}{2} & 0 \quad t \\ 2\left(1 - \frac{j}{g^*(T)}\right)^3 & \frac{g^*(T)}{2} \end{cases}$$

GPH*ta*

$$I_Z(\omega_j) = \frac{1}{2\pi \sum_{t=0}^{T-1} \left(\quad (\;) \right)^2} \left| \sum_{t=0}^{T-1} (\;) \quad (\quad) \right|^2 \quad t$$

$$\varphi \quad t$$

ESTIMATION OF THE DIFFERENCINGPARAMETER OF ARFIMA MODEL BY SPECTRAL REGRESSION METHOD(4)

$$tap(t) = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi(t+0.5)}{T}\right)\right]$$

5 Estimate *d by Ordinary Least Square Method*.

Where,

$$Y_j = \ln I(\omega_j) \quad , \quad X_j = \left| \ \right|^{-2} = \quad j \quad - \quad \omega -$$