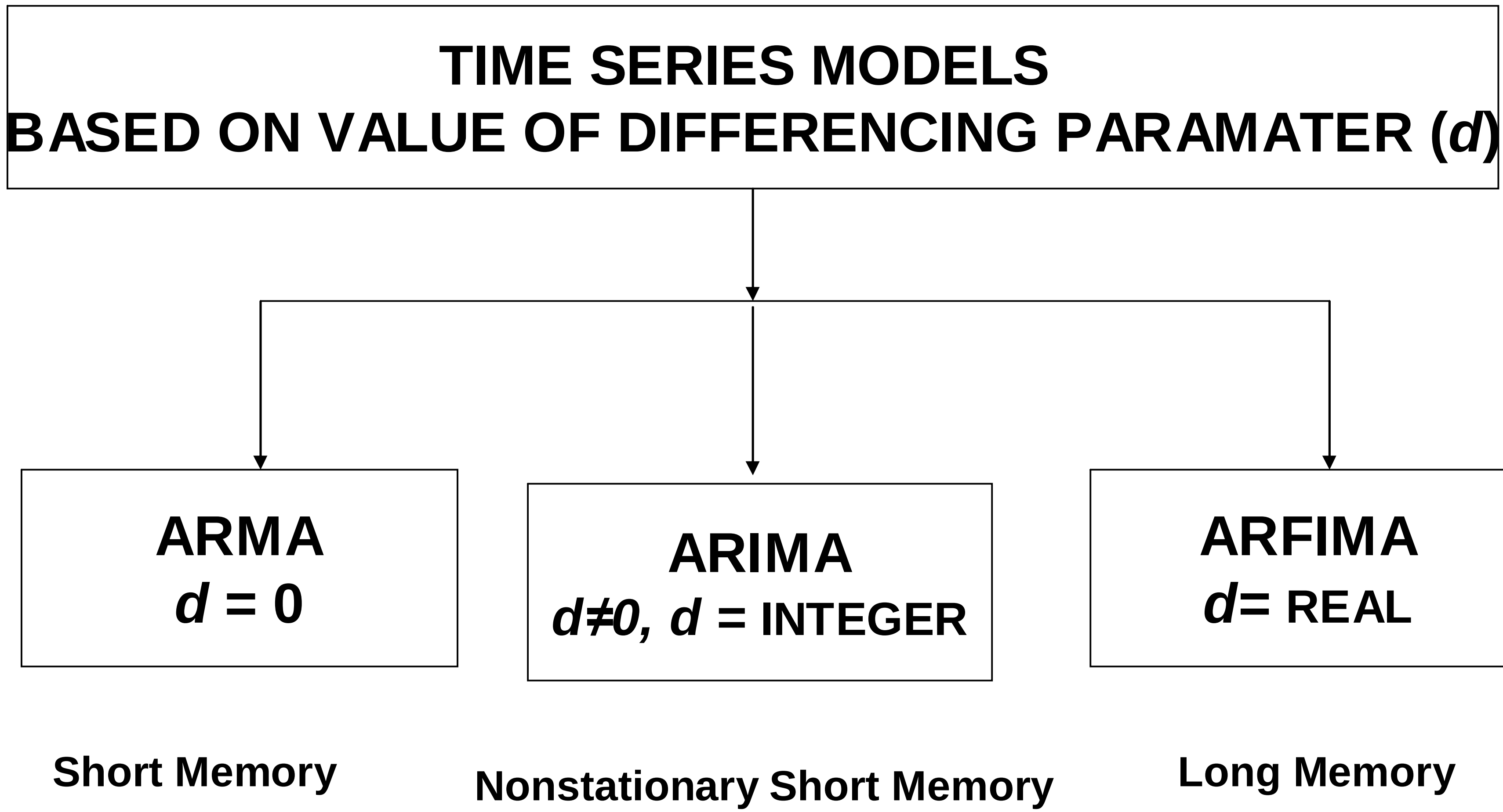
COMPARISON OF DIFFERENCING PARAMETER ESTIMATION FROM NONSTATIONER ARFIMA MODEL BY GPH METHOD WITH COSINE TAPERING

By

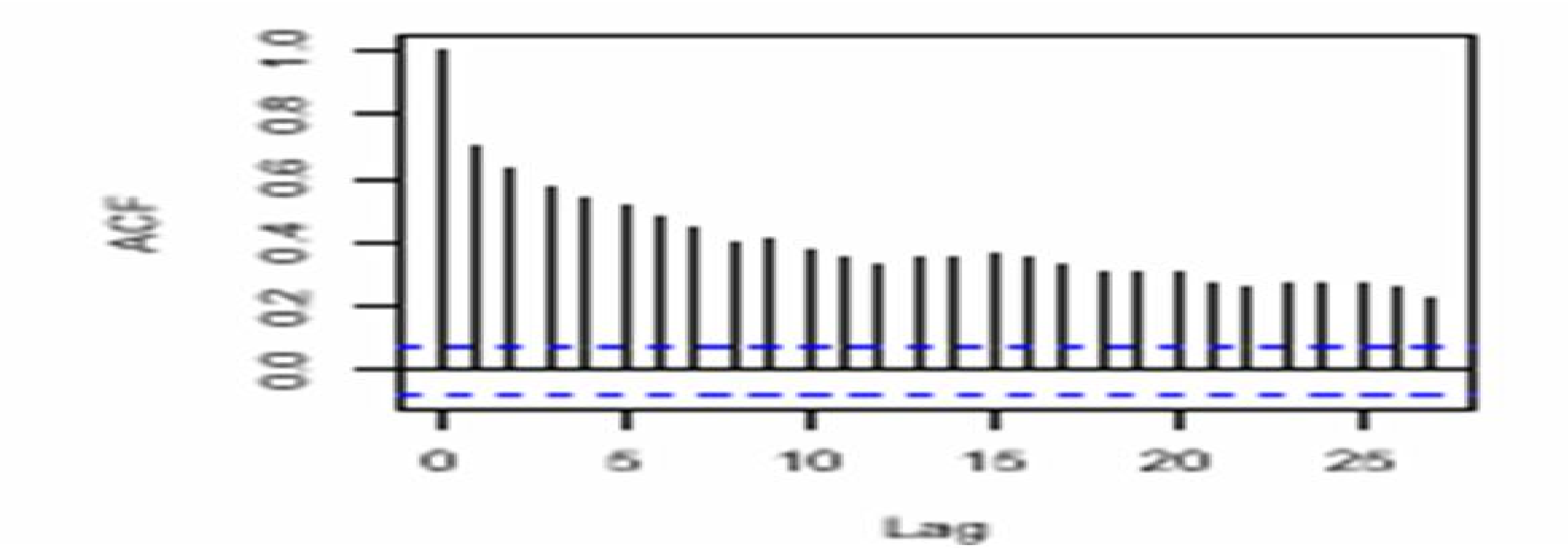
Gumgum Darmawan

INTRODUCTION (1)

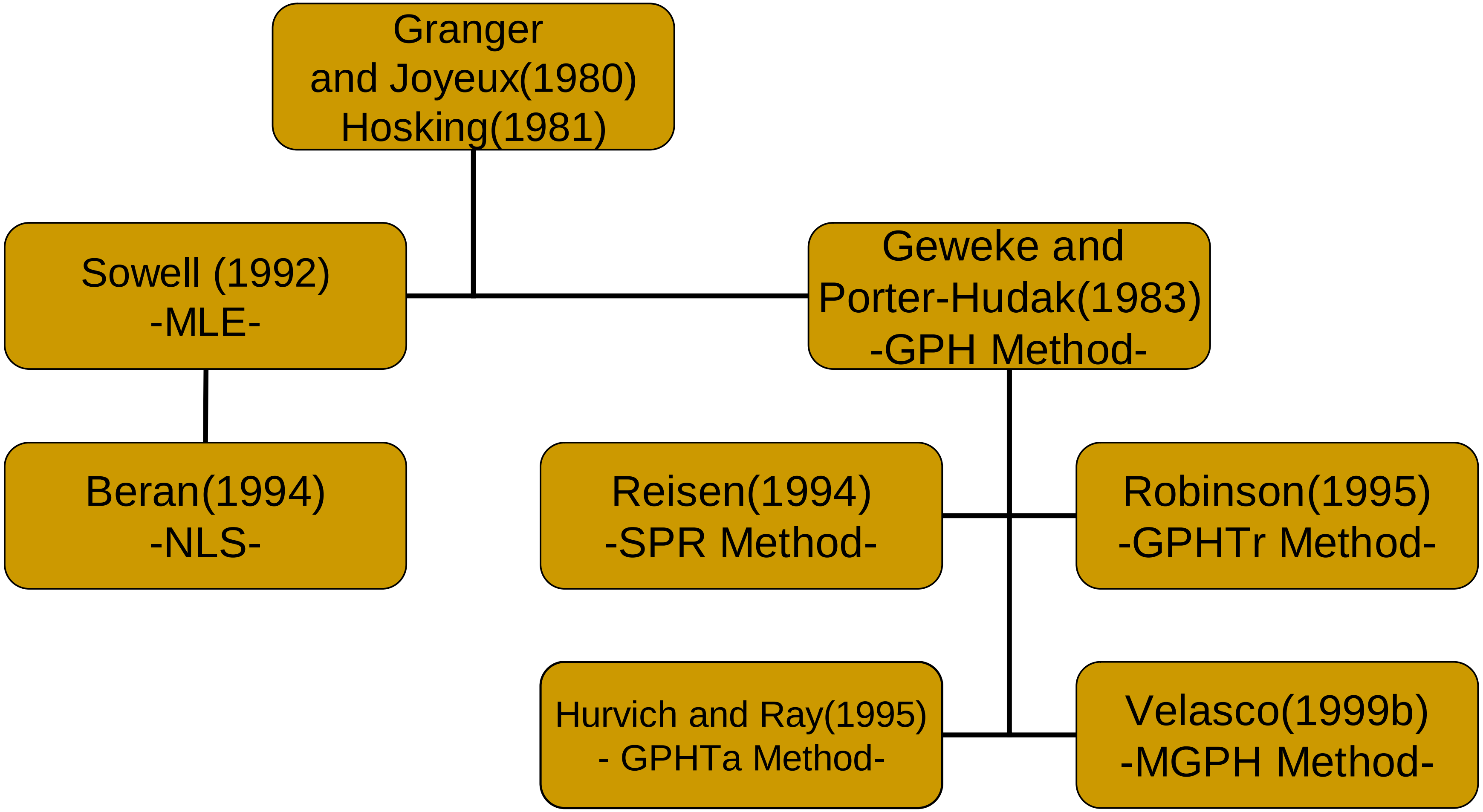


Long Memory Processes

- Long range dependence or long memory means that observations far away from each other are still strongly correlated.
- The correlation of long memory processes is decay slowly as lag data increase that is with a hiperbollic rate.



INTRODUCTION(2)



INTRODUCTION(3)

COMPARISON OF
REGRESSION SPECTRAL
METHODS

Lopes,Olberman
and Reisen(2004)
-Non stationary ARFIMA-
SPR Method is the best

Lopes
and Nunes(2006)
-ARFIMA(0,*d*,0)-
GPH Method is the best

GOAL OF RESEARCH

Comparing accuracy of GPH estimation methods with cosine tapering of the differencing parameter (*d*) and forecasting result from nonstationary ARFIMA Model by Simulation Study.

ARFIMA MODEL(1)

- An ARFIMA(*p,d,q*) model can be defined as foll ows:

$$\phi(B\,(\;-)\,)^{d}(1_t\;)\;\;B\;\;(\;)\,-z\;\;t$$

- t = index of observation (t = 1, 2, . . ., T)
- d = differencing parameter (rea l number)
- μ = mean of obervation

$$a_t \sim NID(0,\sigma^2),$$

ARFIMA MODEL(2)

- $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - .. - \phi_p B^p$ is polinomial AR(*p*)
- $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - .. - \theta_q B^q$ is polinomial MA(*q*)
- $(1-B)^d = \nabla^d = \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k$ fractional differencing operator

$$tap_1(t) = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi(t+0.5)}{T}\right)\right]$$

$$tap_2(t) = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi(t+1)}{T+1}\right)\right]$$

$$tap_3(t) = 0.54 - \left[0.46\cos\left(\frac{2\pi t}{T-1}\right)\right]$$

DIAGRAM

*Generate
Data 1 :
ARFIMA(1,*d*,0)
and
ARFIMA(0,*d*,1)
d =0.6, 0.7 and 0.8
N = 600 and 1000

↓

Estimate parameter ***d*** by GPH Method

1. Cosine Bell Taper
2. Hanning Taper
3. Hamming Taper

↓

Determine Mean dan SD estimate
From these methods

Mean and Standard deviation of parameter estimation *d*

T	ARFIMA Model Data		GPH Estimation Method With Cosine Taper					
			Cosine Bell		Hanning		Hamming	
			d	**sd(*d*)**	***d***	**sd(*d*)**	***d***	**sd(*d*)**
600	**ARFIMA(1,*d*,0)**	*d*=0,6	0,624	0,195	0,620	0,200	0,617	0,195
		d=0,7	0,717	0,196	0,731	0,208	0,728	0,192
		d=0,8	0,840	0,198	0,833	0,193	0,830	0,189
	ARFIMA(0,*d*,1)	*d*=0,6	0,581	0,201	0,570	0,205	0,585	0,192
		d=0,7	0,687	0,198	0,685	0,204	0,696	0,187
		d=0,8	0,791	0,203	0,790	0,201	0,787	0,191
1000	**ARFIMA(1,*d*,0)**	*d*=0,6	0,611	0,172	0.602	0,173	0,6111	0,168
		d=0,7	0,712	0,178	0,705	0,170	0,716	0,197
		d=0,8	0,803	0,179	0,815	0,172	0,834	0,202
	ARFIMA(0,*d*,1)	*d*=0,6	0,598	0,175	0,583	0,174	0,592	0,162
		d=0,7	0,695	0,172	0,677	0,176	0,687	0,187
		d=0,8	0,803	0,167	0,794	0,176	0,783	0,199

MSE of Forecasting (h = 10)

T	ARFIMA Model Data		Cosine Taper		
			Cosine Bell	**Hanning**	**Hamming**
300	**ARFIMA(1,*d*,0)**	*d*=0,6	1.316	1.317	1.311
		d=0,7	1.322	1.322	1.316
		d=0,8	1.342	1.342	1.336
	ARFIMA(0,*d*,1)	*d*=0,6	1.254	1.254	1.251
		d=0,7	1.254	1.255	1.251
		d=0,8	1.232	1.233	1.229
600	**ARFIMA(1,*d*,0)**	*d*=0,6	1.367	1.367	1.360
		d=0,7	1.323	1.323	1.317
		d=0,8	1.352	1.352	1.346
	ARFIMA(0,*d*,1)	*d*=0,6	1.262	1.262	1.258
		d=0,7	1.219	1.219	1.215
		d=0,8	1.633	1.633	1.622

CONCLUSION

1) GPH method with Cosine Bell Tapering shows the best performance in estimating the differencing parameter of ARFIMA(0,*d*,1)
2) From ARFIMA(1,*d*,0) data, GPH method with Hanning Taper is the best estimator all of method.
3) From forecasting result, Mean square Error (MSE) of GPH method with Hamming taper has the least value of all data types.

THANK YOU…..

THANK YOU

ESTIMATION OF THE DIFFERENCING PARAMETER OF ARFIMA MODEL BY SPECTRAL REGRESSION METHOD(1)

1. Construct spectral density function (SDF) of ARFIMA model

$$f_Z(\omega) = \frac{\sigma_a^2}{2\pi}\left|\frac{\theta_q(\exp(-i\omega))}{\phi_p(\exp(-i\omega))}\right|^2 \left\{2\sin\left(\frac{\omega}{2}\right)\right\}^{-2d}, \quad \omega \in [-\pi, \pi], \quad (2)$$

2. Take logarithms of SDF from ARFIMA model

$$\ln f_Z(\omega_j) = \ln f_W(0) + d\ln\left|1-\exp(-i\omega_j)\right|^{-2} + \ln\left(\frac{f_W(\omega_j)}{f_W(0)}\right) \quad (3)$$

where $\omega_j = \frac{2\pi j}{T}$, $j = 1, 2, \ldots, T/2$

ESTIMATION OF THE DIFFERENCING PARAMETER OF ARFIMA MODEL BY SPECTRAL REGRESSION METHOD(2)

3. Add natural logarithm of periodogram to equation (3) above

$$\ln\{I_Z(\omega_j)\} = \ln\{f_W(0)\} - d\ln\left|1-\exp(-i\omega_j)\right|^2 + \ln\left\{\frac{f_W(\omega_j)}{f_W(0)}\right\} + \ln\left\{\frac{I_Z(\omega_j)}{f_Z(\omega_j)}\right\} \quad (4)$$

4. Determine the periodogram based on regression spectral methods

GPH

$$I_Z(\omega_j) = \frac{1}{2\pi}\left\{\gamma_0 + 2\sum_{t=1}^{g(T)} \gamma_t \cos(t.\omega_j)\right\},$$

ESTIMATION OF THE DIFFERENCINGPARAMETER OF ARFIMA MODEL BY SPECTRAL REGRESSION METHOD(3)

GPH*ta*

$$I_Z\left(\omega\right)_j = \frac{1}{2\pi\sum_{t=0}^{T-1}\left(\;\left(\;\right)\right)^2}\left|\sum_{t=0}^{T-1}\left(\;\right)\;\left(\qquad\right)\right|^2 \quad \varphi \;\; t \quad t \quad j$$

$$tap_1(t) = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi\left(t+0.5\right)}{T}\right)\right]$$

$$tap_2(t) = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi\left(t+1\right)}{T+1}\right)\right]$$

$$tap_3(t) = 0.54 - \left[0.46\cos\left(\frac{2\pi t}{T-1}\right)\right]$$

ESTIMATION OF THE DIFFERENCINGPARAMETER OF ARFIMA MODEL BY SPECTRAL REGRESSION METHOD(4)

5 Estimate *d by Ordinary Least Square Method*.

Where,

$$Y_j = \ln I\left(\omega_j\right) \quad , \quad X_j \left| \quad \right|^{-2} = \quad _j - \quad \omega -$$