

# Mathematical Models to Describe Egg Production in Laying Hens (Review)

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## **Abstrack**

This paper is purposed to review the mathematical model to describe the egg production curve in laying hens. The curve of egg production is normally similar between breeds or strains, it increases form first lay to reach the peak at a certain age, and then decreases gradually to the end of the laying period. Many mathematical models to describe the egg production have been published, and they are sufficient to predict the number of egg production a long the age, but for breeding application needs to develop the model for small group population. If the characteristics of sexual maturity are taken into account, the model from Yang is more favourable than other mathematical models.

*Keywords* : *Mathematical Models, Egg Production, Laying Hen*

# Model Matematik untuk Menggambarkan Kurva Produksi Telur pada Ayam Petelur (Review)

## **Abstrak**

Makalah ini bertujuan untuk me-review model matematik yang dapat mendeskripsikan kurva produksi telur pada ayam petelur. Kurva produksi telur umumnya sama, baik untuk bangsa ataupun strain, yaitu meningkat pada awal masa bertelur untuk mencapai puncaknya pada umur tertentu dan akan menurun secara gradual sampai akhir periode bertelur. Banyak model matematik kurva produksi telur yang sudah dipublikasikan, dan pada umumnya model-model tersebut sudah cukup bisa digunakan untuk menduga produksi telur saja, tapi jika untuk keperluan pemuliaan ternak, perlu dikembangkan model yang bisa menduga produksi pada populasi yang kecil. Jika karakteristik kematangan seksual turut dipertimbangkan, maka model Yang lebih menguntungkan jika dibandingkan dengan model-model matematik lainnya.

*Kata kunci* : *Model Matematik, Produksi Telur, Ayam Petelur*

## **Introduction**

Egg production is a result of many genes through biochemical, anatomical, and physiological processes. The amount of egg lied by hens is influenced by many factors, such as breed or strain, age of birds, photorefractoriness, broodiness, moulting, nutrition, and other environmental factors. However, the egg production normally follows a certain curve or the pattern of the curve is usually similar. The number of egg normally increases to reach the peak in certain age, and then decrease to the end of laying periods.

Although the number of egg lied by chicken varies between breeds or strain, the shape of the curve is absolutely similar. Knowing the model to describe the egg production is very essential for animal breeder as it is used to make standard performance of the

commercial strain, sold to the customers. This standard performance is of such a guild line to understand the production.

Many mathematical models to describe the egg production in laying hens have been published. Such of them is just to predict the number of egg produced and other considers the biological characters of the hens. This paper will review directly on the relationship between age and stage of lay.

### **Description of Egg Production**

With an appropriate environment, genes controlling processes associated with egg production express fully genetic potential (Fairfull and Gowe, 1990). In domestic fowl, birds start laying about 21 weeks of age, but the age at first laying or sexual maturity can be delayed by the changes of environment, such as day length during rearing (Rose, 1997). Determination of the sexual maturity in poultry is difficult, but biologically it is indicated by the first ovulation. Thus, first egg laid by a hen expresses the sexual maturity for whole birds (Pingel, *et al.*, 1987). Sexual maturity is specific for each breed, strain or population. The birds with late sexual maturity have generally low egg production (Brandsch, 1981; Pingel, *et al.*, 1987)

Age influences within the laying periods, with the increase in yield form first laying to a peak, and then egg production decreases gradually to the end of the first cycle (Gowe and Fairfull, 1982). Therefore, it is possible to breed the birds in the second cycle, but the yield is usually lower than the first cycle (Bessei, 1988). Rose, (1997) divided the egg laying period into three parts. First period is the time from the first egg laying to the time when the birds lay continuously. Second period is the main laying period and last for various length of time, depending on breed and strain of birds. The third period occurs when the ova shed decline rapidly, indicated by moulting, broody behaviour or changes in nutrient intake. Moulting allows tissues to regenerate. Oviducts regress and their weight reduce about 10%, whereas the ovary decline about 20%. There is also reduction in the other tissues such as fat and body weight. Feathers start loosing about 15 days after the commencement of moulting, and they recover again up to 100 days after moulting (Rose, 1997).

### **Mathematical Models**

Mathematically, the curve of egg production can be divided into 3 stages: (1) the increase in slope from first lay to the peak, (2) the peak, (3) A decrease in slope from the peak to the end of production. The pattern of egg production curve is similar to that of lactation curve, therefore several models were derived from the lactation curves. McNally, (1971), for example, derived the egg production model from Wood model.

There are at least 9 mathematical models of egg production that have been published. The models are presented below:

#### **1. Gamma function (Wood, 1967)**

$$y_t = at^b e^{(-ct)}$$

where  $y_t$  = average of daily yield

$t$  = time (week)

$a, b, c$  = constant

This model was performed for milk production in dairy cattle, but it can be used also to describe the egg production.

**2. Modification of Wood model, applied to poultry (McNally, 1971)**

$$y_t = at^b e^{(-ct+dt^{\frac{1}{2}})}$$

where  $y_t$  = egg production during  $t$   
 $t$  = time  
 $a, b, c, d$  = constant

**3. McMillan function (McMillan, et al., 1970ab)**

$$y_t = M(1 - e^{-\xi(t-t_0)})e^{-\alpha t}$$

where  $y_t$  = egg production during  $t$   
 $M$  = the potential maximum daily egg production  
 $t$  = time  
 $t_0$  = the initial day of egg laying  
 $\xi$  = the rate of increase in egg laying  
 $\alpha$  = the rate of decrease in egg laying

This model was performed to predict egg production in *Drosophila*.

**4. Algebraic function (Adam and Bell, 1980)**

$$y_t = \frac{1}{0.01 + ar^{(t-b)}} - c(t - d)$$

where  $y_t$  = percent of hen day production during  $t$   
 $t$  = time (week)  
 $a, b, c, r, d$  = constant

**5. Compartmental model (McMillan, 1981)**

$$y_t = A(e^{-k_2 t} - e^{-k_1 t})$$

where  $y_t$  = average of egg production during  $t$   
 $t$  = time  
 $k_1, k_2$  = instantaneous rate of increase and decrease in egg production  
 $A$  = maximum potential of egg production

**6. Post-peak of linear regression (Gavora, et al., 1982)**

$$y_t = m - kt$$

where  $y_t$  = average of egg production during  $t$   
 $t$  = time (28 day period)  
 $m, k$  = instantaneous rate of increase and decrease in egg production

**7. Logistic model (Cason and Britton, 1988)**

$$y_t = a(e^{-bt}) \left[ \frac{1}{(1 + c^{c+dt})} \right]$$

where  $y_t$  = egg production during  $t$   
 $t$  = age of flock (week)  
 $a, b, c, d$  = constant

### 8. *Modification Compartmental Model (Yang et al., 1989)*

$$y_t = \frac{ae^{-bt}}{[1 + e^{-c(t-d)}]}$$

where  $y_t$  = percent of hen day production during  $t$   
 $t$  = time (week)  
 $a$  = a scale of parameter  
 $b$  = the rate of decrease in laying ability  
 $c$  = the reciprocal indicator of the variation in sexual maturity  
 $d$  = the mean age of sexual maturity

### 9. *Gloor function (Gloor, 1997)*

$$y_t = LL(1 - e^{-at^b})(e^{ct^d})$$

where  $y_t$  = egg production during  $t$   
 $LL$  = asymptote  
 $t$  = time (week)  
 $a$  = the rate of linear increase in egg laying  
 $b$  = the rate of increase in egg laying  
 $c, d$  = the rate of decrease in laying ability

## Comparison of Models

The accuracy of models (measured by  $R^2$ ), obtained by fitting various length of laying periods are presented in Table 1. The same models when they were fitted to the different sets of data resulted in different values of  $R^2$ . It seems that the application of the models depends mainly on the stages and the length of lay, in which inclusive or exclusive moulting. Fairfull and Gowe, (1990) noted that All models were sufficient to forecast flock income and performance, but for breeding applications, it was desirable to develop a model, reliable to predict egg production for small group, such as dam or sire families.

Parameters of McMillan model changed substantially in a selection experiment with two genetically different strains (Timmermans, 1973) and the fit was not good after the peak because the production curve did not arise again as hens got older (Adams and Bells, 1980). Gavora, *et al.*, (1982) compared 3 models, Wood, compartment (McMillan model), and post-peak linear, on egg production from birds synchronised for sexual maturity. The results indicated that compartmental model was the best fit for both individual and groups of egg production, but for unsynchronised birds, Adam-Bell and logistic models were better than compartmental model (Cason and Britton, 1988).

McMillan, *at al.*, (1986) compared 3 models as those being studied by Gavora, *et al.*, (1982). In conclusion, they suggested that the linear model was better to use due to its simplicity and lower costs of fit. However, it was inappropriate to use the linear model in

the early non-linear part of egg production curve. When the model was intended to serve several purposes, for example to predict full record from part record, the compartmental model was appropriate. Mielenz and Müller, (1991) compared 4 models, linear, exponential, Adam-Bell, and McMillan, fitted to 450 days of laying performance. The result showed that there were no significant differences among 4 models for predicted and observed traits, but when they are fitted to the values from period of 1 to 7 (a period was 28 days), all models were unprecise.

Both the compartmental model and the wood model have the limitations in their mathematical properties when they are applied to the mean egg production in a group of hens on the basis of chronological age. The mathematical structure of compartmental model lacks an inflection point in the initial period before the curve reaches the peak, thus the model is not able to characterise the short period of increase in the first 1 to 2 weeks of lay. As a solution, Yang, *at al.*, (1989) derived a model in which incorporates a component of variation in sexual maturity. This model provides more information about a flock and is able to characterise the whole period of lay. Based on a study by Yang, *at al.*, (1989), this model had not only theoretical advantages over the compartmental model and wood model, but also had a better fit.

## **Conclusion**

The curve of egg production is normally similar between breeds or strains, it increases from first lay to reach the peak at a certain age, and then decreases gradually to the end of the laying period. Many mathematical models to describe the egg production have been published, and they are sufficient to predict the number of egg production a long the age, but for breeding application needs to develop the model for small group population. If the characteristics of sexual maturity are taken into account, the model from Yang is more favourable.

Table 1. The accuracy of egg production models, fitted to various length of laying periods

Models	R <sup>2</sup>	Data	Authors
1. $y_t = at^b e^{(-ct)}$ (Gamma function)	0.86-0.96	Interval for 24 fortnights at first laying year	McNally, 1971
	0.95	28 day period started from the first egg	Gavora, <i>at al.</i> , 1982
	0.83-0.93	50 weeks egg production	McMillan, <i>at al.</i> , 1986
	0.69 (hen day) 0.82 (hen housed)	21-72 weeks of egg	Yang, <i>at al.</i> , 1989
2. $y_t = at^b e^{(-ct+dt^{\frac{1}{2}})}$ (Modification of Wood model)	0.94-0.99	Interval for 24 fortnights at first laying year	McNally, 1971
3. $y_t = M(1 - e^{-\xi(t-t_0)})e^{-\alpha t}$ (McMillan function)	0.73-0.95	Regardless of the time scale	Gavora, <i>at al.</i> , 1971
	0.95	28 day period started from the first egg	Gavora, <i>at al.</i> , 1982
	0.97-0.99	50 weeks egg production	McMillan, <i>at al.</i> , 1986
	0.95	First cycle	Cason and Britton, 1988
	0.95 (hen day) 0.95 (hen housed)	21-72 weeks of egg	Yang, <i>at al.</i> , 1989
	0.98	up to 450 days	Mielenz and Müller, 1991
4. $y_t = \frac{1}{0.01 + ar^{(t-b)}} - c(t-d)$ (Algebraic function)	0.99	First cycle	Cason and Britton, 1988
	0.98	up to 450 days	Mielenz and Müller, 1991

Table 1. Continued

Models	R <sup>2</sup>	Data	Authors
5. $y_t = A(e^{-k_2t} - e^{-k_1t})$ (Compartmental model)	0.70-0.85	Weekly egg production during a 30 weeks	McMillan, 1971
6. $y_t = m - kt$ (Post-peak of linear regression)	0.91	28 day period started from the first egg	Gavora, <i>at al.</i> , 1982
	0.98-0.99	50 weeks egg production	McMillan, <i>at al.</i> , 1986
	0.93	up to 450 days	Mielenz and Müller, 1991
7. $y_t = a(e^{-bt})\left[\frac{1}{(1+c^{c+dt})}\right]$ (Logistic model)	0.99	First cycle	Cason and Britton, 1988
8. $y_t = \frac{ae^{-bt}}{[1+e^{-c(t-d)}]}$ (Modification Compartmental Model)	0.98 (hen day) 0.99 (hen housed)	21-72 weeks of egg	Yang, <i>at al.</i> , 1989
9. $y_t = LL(1 - e^{-at^b})(e^{ct^d})$ (Gloor function)	1.00	up to 80 weeks without moulting	Gloor, 1997

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